CHAPTER 32 Inductance and Circuit Oscillations

Answers to Understanding the Concepts Questions

1. The principle that governs the existence of mutual inductance is a general one. However, it is possible to arrange the circuits so that quantity M, the mutual inductance itself, is zero. Because the orientation of the two circuits is usually not time dependent, this means arranging the circuits so that the magnetic flux from one does not link the other. For example, two sma1l planar loops are only weakly linked if they are so oriented that their planes are perpendicular.

2. A coil has more self inductance than a straight wire. Initially, as the current in the coil increases, more work must be dine to overcome the induced emf that opposes the increase in the magnetic flux through the coil. This work is converted into the magnetic energy stored in the coil. Once the current in the coil reaches a constant value, however, no emf is induced any longer, and the work needed in driving a current through it simply equals the thermal energy dissipated on the wire, and that is proportional to R, its resistance. Bending the wire does not change R, so once the current no longer changes it makes no difference whether the wire is straight or coiled.

3. (b) will have the largest mutual inductance; the magnetic field generated by one coil passes almost directly through the other. (a) will have the next-largest mutual inductance; if we think of the coil as generating a field like that of a permanent magnet, the magnetic field from one comes back around to close on itself; and in doing so passes through the other. However, the field is more "spread," so the linking is weaker. (c) will have the weakest linking; for example, the field comes up through the lower coil but no flux from it passes through the upper coil (see the response above to question 1).

4. An appliance that consumes a large amount of power needs a large current. As the AC current flows through the appliance the inductance of the appliance generates a large backward emf that might significantly weaken the overall emf that powers the circuit, therefore dimming the lights.

5. Yes. It is direct to calculate the magnetic flux passing through the loop due to the magnetic field generated around the straight wire. This flux will be proportional to the current in the straight wire, and hence gives the mutual inductance. By the "mutual" property of the mutual inductance, that also tells us the emf generated in the straight wire. Where is the loop associated with the straight wire? That wire must close somewhere, and that is what forms the loop of which the straight wire is a part.

6. Equate the electric and magnetic energy densities: $\varepsilon_0 E_{\max}^2/2 = B_{\max}^2/2\mu_0$, or $E_{\max} = (1/\varepsilon_0\mu_0)B_{\max} = cB_{\max}$. Plug in the values ε_0 of and μ_0, and check out the value of $c = (1/\varepsilon_0\mu_0)$ for yourself. (Hint: the unit of this combination turns out to be that of speed. And this is no ordinary speed!) The fact that ε_0 of and μ_0, two fundamental constants coming from very different origins, come together here in a unique way to produce this universal speed, is of profound significance in physics. The detailed analysis will be made in Chapter 34.

7. Let $Q = Q_0\,e^{-\alpha t}\sin\omega t$, then $I = dQ/dt = -\alpha Q_0\,e^{-\alpha t}\sin\omega t + \omega Q_0\,e^{-\alpha t}\cos\omega t = I_0\,e^{-\alpha t}\sin(\omega t + \phi_0)$, so the amplitude of I decays with the same exponential factor as that of Q. The answer is then (b).

8. A spark is the result of a brief, but ultra-high, spike in electric potential difference that causes the air to ionize. Such spikes can result from a rapid change in the current flowing in the circuit, as the induced emf is proportional to dI/dt. When you turn the light switch on, the current in the circuit cannot abruptly jump from zero to its finite value, as the backward emf caused by the inductance of the circuit prevents this from happening. So dI/dt cannot be too large, and generally no spark is generated. When you turn off the switch, however, you are terminating the current flow immediately, and the current rapidly drops to zero. The large corresponding value of dI/dt can result in a voltage spike high enough to produce a spark.

9. You would have to be able to concoct a switch in addition to the items listed. You could then arrange the items in an RL circuit of the type shown in Fig 32-10. The time dependence of the rise in the potential across the resistor or the inductor would allow us to solve for the value of the inductance. If L were too small compared to R, you would have to exercise more ingenuity in the switch construction and in the use of your timer, because the exponential rise (or fall) time R/L would be too small to observe with your unaided senses.

10. When the switch is closed there is an induced emf on the inductor, in the amount of $L(dI/dt)$. With a large L value and a rapidly changing current, this emf, even though brief, is very significant. Being parallel to the lightbulb, this emf is applied on the lightbulb, causing it to flash momentarily. Once the current stabilizes, however, the induced emf drops to zero and the current largely bypasses the lightbulb to go through the inductor of low resistance, and the lightbulb glows dimly. As the switch is opened, there is again a brief surge in the value of dI/dt, and the resulting large induced emf in the inductor once again drives a large current through the lightbulb, causing it to flash before going out.

11. It flows in the fields as well as in actual movement of charge. We have already seen that there is energy in electric and magnetic fields. The energy in the inductor is purely magnetic while the energy in the capacitor is purely electric. The exchange takes place as the current flows; the movement of charges will generate both electric and magnetic fields. And of course a moving set of charges carries kinetic energy.

12. If we place the range of the capacitance values of common capacitors to be between 1 pF and 1 F, and that of the inductance of common inductors to be between 0.1 mH and 10 H, then from $f = (1/2\pi)(LC)^{-1/2}$ we find the corresponding frequency range of electronic oscillators to be approximately between 0.1 Hz and 10^7 Hz.

13. It depends on the total electromagnetic energy U in the circuit: $U = \frac{1}{2}Q_{max}{}^2/C$. Also, note that $U = \frac{1}{2}LI_{max}{}^2$, so $Q_{max} = (LC)^{1/2}\, I_{max} = I_{max}/\omega$.

14. Suppose we wind the wire into an N-turn solenoid of length l and cross-sectional area A: $L = \mu_0 A l n^2 = \mu_0(\pi r^2)l(N/l)^2 = \text{constant} \times r^2N^2/l$, with r the cross-sectional radius. But the total length l_w of the wire is fixed, i.e., $l_w = 2\pi r l$, so $r = l_w/2\pi l$. Also, if the thickness of each turn of the wire is t then $l = Nt$. Thus $L = \text{constant} \times\ l_w^{\,2}/Nt^3$, which is proportional to $1/N$. This analysis suggests that the self inductance if the wire is maximized if we simply bend it into a single-turn, circular loop. (This analysis, however, must be carefully re-examined since the self-inductance of a single circular loop does not follow the simple formula for a long solenoid by taking $N = 1$; it involves finding the magnetic flux of a non-uniform field distribution and the integral involved is not trivial. Our analysis, if anything, does suggest that shortening the length of the solenoid would increase L, at least within a certain range of N.)

15. Yes, with an energy density proportional to the field squared at any given location. In the limit that the solenoid is ideal, this field becomes zero and the magnetic energy density outside disappears. For a real solenoid, even one that is close to ideal, the energy density outside may be small, but the integral of that energy over all space is not negligible.

16. The magnetic field expression used in the calculation of the magnetic energy density is Eq. (32-8), that of an ideal solenoid. An ideal solenoid is infinitely long, and the magnetic field lines inside the solenoid are high concentrated within its finite cross-section. As the field lines close up the loop outside the solenoid, however, they each form an infinite arc. The vast space in which these field lines occupy outside the solenoid greatly "dilutes" their concentration, so the field is virtually zero outside and does not contribute to the magnetic energy density.

17. The current in an RL circuit without a power supply decreases over time as the magnetic energy stored in the inductor is gradually dissipated into heat on the resistor. With $R = 0$ there is no mechanism for this energy dissipation, so the magnetic energy, and hence the current in the inductor, persist forever.

18. A diamagnetic core responds to the magnetic field of the current in the solenoid with an opposite field, so the net field, and therefore the magnetic flux, decreases. The inductance, which is proportional to the magnetic flux, must also decrease. The answer is (b).

19. The inductive and capacitive energies of the RLC circuit are analogous, respectively, to the kinetic and potential energies of the mechanical spring. This can be seen directly from Table 32-3, which shows that $LI^2/2$ is analogous to $m(\mathrm{d}x/\mathrm{d}t)^2/2$ and $Q^2/2C$ corresponds to $kx^2/2$. Energy flows back and forth between kinetic and potential terms in the damped mechanical spring as it does between inductor and capacitor in the RLC circuit. In each case there is in addition a mechanism for energy loss.

20. In obtaining Eq. (32-13), we considered $\mathrm{d}U_L$, the change in magnetic energy stored in the inductor, as the work done by the power supply in overcoming the backward emf due to the self-inductance of the inductor: $\mathrm{d}U_L = \mathcal{E}_L I\,\mathrm{d}t = L(\mathrm{d}I/\mathrm{d}t)I\,\mathrm{d}t = LI(\mathrm{d}I/\mathrm{d}t)$, which, upon integration, yields Eq. (32-13). With another coil nearby, we must consider not only $\mathcal{E}_L$, but also $\mathcal{E}_M$, the emf due to the mutual inductance between the two circuits. To this end we may write

$$\mathrm{d}U_{L1} = (\mathcal{E}_{L1} + \mathcal{E}_{M1})\, I_1\mathrm{d}t = [L_1(\mathrm{d}I_1/\mathrm{d}t) + M(\mathrm{d}I_2/\mathrm{d}t)]\, I_1\,\mathrm{d}t \quad \text{(for circuit 1);}$$
$$\mathrm{d}U_{L2} = (\mathcal{E}_{L2} + \mathcal{E}_{M2})\, I_2\mathrm{d}t = [L_2(\mathrm{d}I_2/\mathrm{d}t) + M(\mathrm{d}I_1/\mathrm{d}t)]\, I_2\,\mathrm{d}t \quad \text{(for circuit 1).}$$

21. If we think of regions with higher (lower) net field as regions of higher (lower) pressure, then the region between two parallel wires carrying current in the same direction is a region of lower pressure, because the contributions from each wire to the field in that region between them tend to cancel whereas the contributions in the regions to the outside of both wires tends to add. Hence the attraction is expressed as the influence of higher external and lower internal pressure. If the wires carry current in opposite directions, the fields tend to add in the region between the wires and cancel in the region outside the two wires, so a net internal pressure tends to push the wires apart. This analogy can be carried farther by thinking about the forces between current sheets, in which case you can show that the magnetic forces are perpendicular to the surfaces of the sheets and proportional to the area, just like a pressure. This idea is heavily used in magnetohydrodynamics, which concerns electrically conducting fluids and is applicable to many plasma physics problems.

22. With the two currents flowing in the same direction, the magnetic fields generated by the two currents point in opposite directions in the region in between the two currents and in the same direction outside the region. So the magnetic energy density, which is proportional to B^2, is weakened in the region in between the two currents due to the superposition of their magnetic fields and enhanced outside that region. The fact that the magnetic force between the two currents flowing in the same direction is attractive can be interpreted as the tendency of the magnetic force to push a current-carrying wire from a region of higher magnetic energy density to a region of lower energy density. A similar analysis for the case when the two currents are opposite in direction also points to the same tendency, only this time the region in between the two currents has an enhanced energy density and the force is now repulsive.

Solutions to Problems

1. We find the magnetic flux from

$$\Phi_B = LI = (2 \times 10^{-3}\ \text{H})(30 \times 10^{-3}\ \text{A}) = \boxed{6.0 \times 10^{-5}\ \text{Wb}}.$$

2. We use the expression for the inductance of a solenoid,

$L = \mu_0 A \ell n^2$, which gives

$$\boxed{L/A\ell = \mu_0 n^2}.$$

3. Because there is a magnetic field only inside the solenoid, the magnetic flux through the single loop is

$$\Phi_{B,\text{loop}} = B_{\text{solenoid}} A_{\text{solenoid}} = \mu_0 n I_{\text{solenoid}} A_{\text{solenoid}}.$$

We find the mutual inductance from

$\Phi_{B,\text{loop}} = M I_{\text{solenoid}}$, which gives

$$M = \mu_0 n A_{\text{solenoid}}$$
$$= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})[(600\ \text{turns})/(0.25\ \text{m})]\pi(1.8 \times 10^{-2}\ \text{cm})^2 = 3.1 \times 10^{-6}\ \text{H}$$
$$= \boxed{3.1\ \mu\text{H}}.$$

4. We use the inductance of a solenoid in the expression for the magnitude of the induced emf:

$$\mathcal{E} = L\ \text{d}I/\text{d}t = \mu_0 A \ell n^2\ \text{d}I/\text{d}t = (\mu_0 A\, N^2/\ell)\ \text{d}I/\text{d}t;$$

$0.520\text{V} = [(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.25 \times 10^{-2}\ \text{m})^2 N^2/(12 \times 10^{-2}\ \text{m})](1.25\ \text{A/s})$, which gives

$$N = \boxed{4.5 \times 10^4\ \text{turns}}.$$

5. We find the self-inductance from

$$\mathcal{E} = -L\ \text{d}I/\text{d}t;$$

$-0.3\ \text{V} = -L(10\ \text{A/s})$, which gives $L = 0.03\ \text{H} = \boxed{30\ \text{mH}}$.

6. Differentiating the given current, $I = I_0 \cos(\omega t)$, we have

$$\mathcal{E} = -L\ \text{d}I/\text{d}t = +L\omega I_0 \sin(\omega t).$$

The maximum emf is

$$\mathcal{E}_{\max} = L\omega I_0;$$

$150 \times 10^{-3}\ \text{V} = L(2.7 \times 10^2\ \text{rad/s})(0.60\ \text{A})$, which gives $L = 9.3 \times 10^{-4}\ \text{H} = \boxed{0.93\ \text{mH}}$.

$\boxed{7}$. (a) We find the mutual inductance from

$$\Phi_B(1) = \Phi_{B1} + M_{12} I_2;$$

$0.012\ \text{T}\cdot\text{m}^2 = 0.010\ \text{T}\cdot\text{m}^2 + M_{12}(2\ \text{A})$, which gives $M_{12} = \boxed{1.0\ \text{mH}}$.

(b) Because $M_{21} = M_{12}$, the flux through the second circuit is

$$\Phi_B(2) = L_2 I_2 + M_{21} I_1$$
$$= (1 \times 10^{-3}\ \text{H})(2\ \text{A}) + (1.0\ \text{mH})(1\ \text{A}) = \boxed{3 \times 10^{-3}\ \text{Wb}}.$$

8. For two inductors placed in series, the current through each inductor is the same. This current is also the current through the equivalent inductor, so the total emf is

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$
$$= (-L_1\ \text{d}I/\text{d}t) + (-L_2\ \text{d}I/\text{d}t) = -(L_1 + L_2)\ \text{d}I/\text{d}t = -L_{\text{eq}}\ \text{d}I/\text{d}t,$$ which gives

$$\boxed{L_{\text{eq}} = L_1 + L_2}.$$

9. (a) When only one coil is used, we use the expression for the inductance of a solenoid,

$L_1 = \mu_0 A \ell n_1^2 = \boxed{\mu_0 A N_1^2/\ell}$; or

$L_2 = \mu_0 A \ell n_2^2 = \boxed{\mu_0 A N_2^2/\ell}$, depending on which one has a closed circuit.

(b) If the series windings are in the same direction, the magnetic fields of the two coils are in the same direction, so the total magnetic field is

$B_{\text{total}b} = \mu_0(n_1 + n_2)I = \mu_0(N_1 + N_2)I/\ell$.

Because this field passes through all the turns of both coils, we have

$\Phi_{Bb} = B_{\text{total}b} A(N_1 + N_2) = \mu_0 A(N_1 + N_2)^2 I/\ell$, so the self inductance is

$L_b = \Phi_{Bb}/I = \boxed{\mu_0 A(N_1 + N_2)^2/\ell}$.

(c) If the series windings are in opposite directions, the magnetic fields of the two coils are in opposite directions, so the total magnetic field is

$B_{\text{total}c} = \mu_0(n_1 - n_2)I = \mu_0(N_1 - N_2)I/\ell$.

Because this field passes through all the turns of both coils and the induced emfs will be in opposite directions, we have

$\Phi_{Bc} = B_{\text{total}c} A(N_1 - N_2) = \mu_0 A(N_1 - N_2)^2 I/\ell$, so the self inductance is

$L_c = \Phi_{Bc}/I = \mu_0 A(N_1 - N_2)^2/\ell$.

(d) The magnetic field from one coil is

$B_1 = \mu_0 n_1 I = \mu_0 N_1 I/\ell$.

Because this field passes through all the turns of the other coil, we have

$\Phi_{B21} = B_1 A N_2 = \mu_0 A N_1 N_2 I/\ell$, so the mutual inductance is

$M = \Phi_{B21}/I = \boxed{\mu_0 A N_1 N_2/\ell}$.

10. For two inductors placed in parallel, the potential difference across each inductor, which is the emf, is the same:

$\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2 = -L_1\, dI_1/dt = -L_2\, dI_2/dt = -L_{\text{eq}}\, dI/dt$.

The total current through the equivalent inductor is

$I = I_1 + I_2$, so we have

$dI/dt = dI_1/dt + dI_2/dt$;

$-\mathcal{E}/L_{\text{eq}} = -\mathcal{E}/L_1 - \mathcal{E}/L_2$, which gives $1/L_{\text{eq}} = (1/L_1) + (1/L_2)$ or $\boxed{L_{\text{eq}} = L_1 L_2/(L_1 + L_2)}$.

11. The flux produced by the varying currents through the coils will be through each coil, so we have

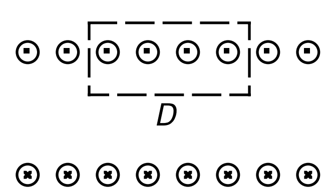
$d\Phi_{B1}/dt = d\Phi_{B2}/dt = d\Phi_B/dt$.

The induced emfs in the coils will be

$\mathcal{E}_1 = -N_1\, d\Phi_B/dt$, and $\mathcal{E}_2 = -N_2\, d\Phi_B/dt$.

Unless $N_1 = N_2$, we have $\mathcal{E}_1 \neq \mathcal{E}_2$. This difference in emf will create an internal current, limited only by the resistance of the coils, so the coils should not be connected in parallel. This is similar to the situation of two different batteries connected in parallel; one will charge the other.

12. We apply Ampere's law to the rectangular path shown in the figure:



$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$.

For a solenoid with a length much larger than the cross-sectional measure, the magnetic field will be zero outside, perpendicular to the segments of the path that pass through the windings, and constant along the segment inside the solenoid:

$\oint \vec{B}\cdot d\vec{s} = \int_{\text{inside}} \vec{B}\cdot d\vec{s} = BD = \mu_0 N_{\text{enclosed}} I$, which gives $B = \mu_0 n I$.

This result is independent of the dimensions of the path, so the magnetic field is the same anywhere inside the solenoid. The total magnetic flux through the coils of the solenoid is

$\Phi_B = NBA = \mu_0 n^2 I A \ell$, so the inductance is $L = \mu_0 n^2 A \ell$.

13. We find the mutual inductance of the solenoid and ring from the flux of the solenoid that passes through the ring:

$$M = \Phi_{B1}/I_1$$
$$= B_1 A_2/I_1 = \mu_0 n I_1 A_2/I_1 = \mu_0 n A_2$$
$$= (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})[(120\ \text{turns})/0.20\ \mathrm{m})]\pi(0.75 \times 10^{-2}\ \mathrm{m})^2$$
$$= 1.3 \times 10^{-7}\ \mathrm{H} = 0.13\ \mu\mathrm{H}.$$

The induced emf in the ring is

$\mathcal{E} = -M\, \mathrm{d}I_1/\mathrm{d}t$, so the current in the ring is

$$I_2 = \mathcal{E}/R = -(M/R)\, \mathrm{d}I_1/\mathrm{d}t.$$

When $t < 0$, there is no change in flux through the ring, so

$I_2 = \boxed{0}$ for $t < 0$.

When $0 < t < 0.30$ s, the induced current is

$$I_2 = -(M/R)\, \mathrm{d}I_1/\mathrm{d}t$$
$$= -[(1.3 \times 10^{-7}\ \mathrm{H})/(33\ \Omega)][(30\ \mathrm{A/s})/(0.30\ \mathrm{s})] = -4.0 \times 10^{-7}\ \mathrm{A} = \boxed{-0.40\ \mu\mathrm{A}} \text{ for } 0 < t < 0.30\ \mathrm{s}.$$

When $0.30\ \mathrm{s} < t < 0.60$ s, the induced current is

$$I_2 = -(M/R)\, \mathrm{d}I_1/\mathrm{d}t$$
$$= -[(1.3 \times 10^{-7}\ \mathrm{H})/(33\ \Omega)][(-30\ \mathrm{A/s})/(0.30\ \mathrm{s})] = +4.0 \times 10^{-7}\ \mathrm{A} = \boxed{+0.40\ \mu\mathrm{A}} \text{ for } 0.30\ \mathrm{s} < t < 0.60\ \mathrm{s}.$$

When $t > 0.60$ s, there is no change in flux through the ring, so

$I_2 = \boxed{0}$ for $t > 0.60$ s.

14. The second solenoid is completely inside the first solenoid. We find the mutual inductance of the two solenoids from the flux of the first solenoid that passes through the second solenoid:

$$\Phi_{B21} = B_1 N_2 A_2 = \mu_0 n_1 I_1 N_2 A_2 = \mu_0 n_1 n_2 \ell_2 \pi R_2^2 I_1.$$

The mutual inductance is

$$M = \Phi_{B21}/I_1 = \boxed{\mu_0 n_1 n_2 \ell_2 \pi R_2^2}.$$

15. The magnitude of the rate of change of the current is the slope:

$$|\mathrm{d}I/\mathrm{d}t| = (0.50\ \mathrm{A})/[(0.45\ \mathrm{s})/4] = 4.4\ \mathrm{A/s},$$

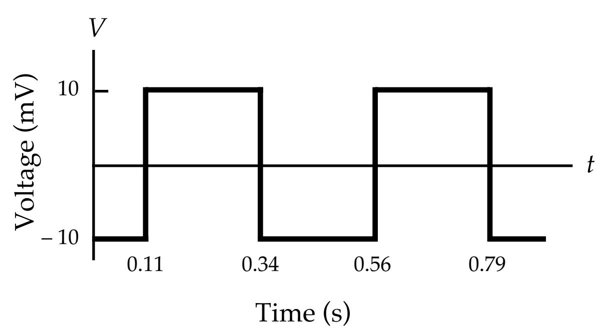
so the magnitude of the induced emf is

$$|\mathcal{E}| = L\, |\mathrm{d}I/\mathrm{d}t| = (2.3\ \mathrm{mH})(4.4\ \mathrm{A/s}) = 10\ \mathrm{mV}.$$

When the slope is positive, the emf will be negative. When the slope is negative, the emf will be positive. Thus we have

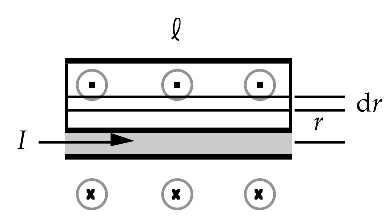
$\mathcal{E} = \boxed{-10\ \mathrm{mV}}$ for $0 < t < T/4$, $3T/4 < t < 5T/4$, ...;

$\mathcal{E} = \boxed{+10\ \mathrm{mV}}$ for $T/4 < t < 3T/4$, $5T/4 < t < 7T/4$, ...



16. When there is a current I in the central wire, the magnetic field in the region between the cylinders is the same as that of a long, straight wire. Because the magnetic field is not constant, we find the magnetic flux between the two conductors by integration. For a differential element we choose a strip of length ℓ at a radius r with width dr:

$$\Phi_B = \iint \vec{B} \cdot \mathrm{d}\vec{A} = \int_{r_0}^{r_1} \frac{\mu_0 I}{2\pi r} \ell\, \mathrm{d}r = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{r_1}{r_0}\right).$$



The self-inductance is

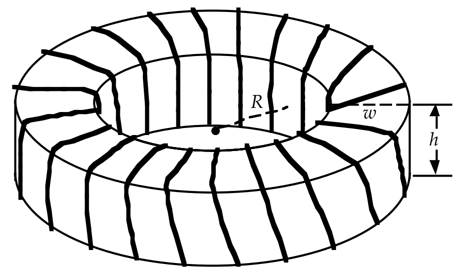
$$L = \Phi_B/I = (\mu\ell/2\pi)\ln(r_1/r_0).$$

17. Because the windings of the two solenoids are in the same direction, the induced emfs will be in the same direction. The total emf in the system is

$$\begin{aligned}\mathcal{E} &= \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_{12} + \mathcal{E}_{21}\\ &= -(L_1\,\mathrm{d}I/\mathrm{d}t) - (L_2\,\mathrm{d}I/\mathrm{d}t) - (M\,\mathrm{d}I/\mathrm{d}t) - (M\,\mathrm{d}I/\mathrm{d}t)\\ &= -L_{\text{eq}}\,\mathrm{d}I/\mathrm{d}t, \text{ which gives}\end{aligned}$$

$$L_{\text{eq}} = L + L + M + M = 2(L + M).$$

18. From the symmetry of the torus, the magnetic field is circular. We apply Ampere's law to a circular path to find the magnetic field inside the torus:



$$\oint \vec{B}\cdot \mathrm{d}\vec{s} = \mu_0 I_{\text{enclosed}};$$

$$B2\pi r = \mu_0 NI, \text{ which gives } B = \mu_0 NI/2\pi r.$$

To find the magnetic flux through one turn of the torus, we integrate over the rectangular cross-section. For a differential element, we choose a vertical strip at a radius r with width dr:

$$\Phi_B = \iint \vec{B}\cdot \mathrm{d}\vec{A} = \int_R^{R+w} \frac{\mu_0 NI}{2\pi r} h\,\mathrm{d}r = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{R+w}{R}\right).$$

The flux through the entire toroidal winding is N times this, so the self-inductance is

$$L = N\Phi_B/I = \boxed{(\mu_0 N^2 h/2\pi)\ln(1 + w/R)}.$$

If $w/R << 1$, we have

$$L = (\mu_0 N^2 h/2\pi)(w/R) = \mu_0 N^2 hw/2\pi R.$$

The cross-sectional area is $A = hw$. The circumference of the torus is $\ell = 2\pi R$, so the density of turns is $n = N/2\pi R$. When we substitute these, we get

$$L = \boxed{\mu_0 n^2 2\pi RA = \mu_0 n^2 A\ell, \text{ which is the self-inductance of a solenoid}}.$$

19. (a) We use the result from Problem 18:

$$L_0 = (\mu_0 N^2 h/2\pi)\ln[1 + (w/R)]$$

$$= [(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(1650\ \text{turns})^2(5.0\times 10^{-2}\ \mathrm{m})/2\pi]\ln\{1 + [(5.0\ \mathrm{cm})/(32.5\ \mathrm{cm})]\} = \boxed{3.9\times 10^{-3}\ \mathrm{H}}.$$

(b) The presence of the core changes the permeability:

$$L = (\mu/\mu_0)L_0 = (4200)(3.9\times 10^{-3}\ \mathrm{H}) = \boxed{16\ \mathrm{H}}.$$

20. (a) Because the current changes uniformly, we have

$$\mathcal{E}_0 = -L\,\mathrm{d}I/\mathrm{d}t = -L\,\Delta I/\Delta t = -(15\times 10^{-3}\ \mathrm{H})(0.120\ \mathrm{A})/(0.50\ \mathrm{s}) = \boxed{-3.6\times 10^{-3}\ \mathrm{V}}.$$

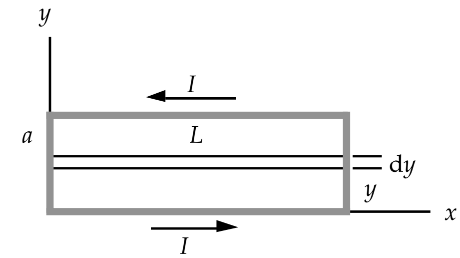
(b) The presence of the core increases the self-inductance and thus the emf:

$$\mathcal{E} = (\mu/\mu_0)\mathcal{E}_0 = (3400)(-3.6\times 10^{-3}\ \mathrm{V}) = \boxed{-12\ \mathrm{V}}.$$

$\boxed{21}$. Because $a << L$, we can ignore the magnetic field from the short sides. The fields from the two long sides are in the same direction, so we can double the field from one when we calculate the flux through the circuit. For the field produced by the bottom wire of radius r, we have

$$B = \mu_0 I y/2\pi r^2, \quad y < r; \quad \text{and} \quad B = \mu_0 I/2\pi y, \quad y \geq r.$$

To find the flux through the loop, we choose a strip dy a distance y from the wire:



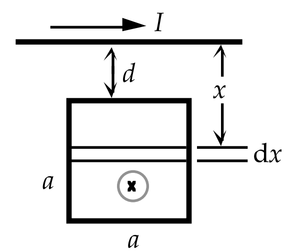
$$\Phi_B = 2\iint \vec{B} \cdot d\vec{A} = 2\int_0^r \frac{\mu_0 I \ell}{2\pi r^2} y\, dy + 2\int_r^a \frac{\mu_0 I \ell}{2\pi y}\, dy = \frac{\mu_0 I \ell}{\pi}\left[\tfrac{1}{2} + \ln\left(\tfrac{a}{r}\right)\right].$$

The self inductance is

$$L = \frac{\mu_0 \ell}{\pi}\left[\tfrac{1}{2} + \ln\left(\tfrac{a}{r}\right)\right].$$

If $r \to 0$, $\ln(a/r) \to \infty$. The radius of wire cannot be neglected.

22. We find the mutual inductance of the system by finding the mutual inductance of the loop. The magnetic field of the wire depends on the distance from the wire. To find the magnetic flux through the loop, we choose a strip a distance x from the wire with width dx:



$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \int_d^{d+a} \frac{\mu_0 I}{2\pi x} a\, dx = \frac{\mu_0 I a}{2\pi} \ln\left(1 + \frac{a}{d}\right).$$

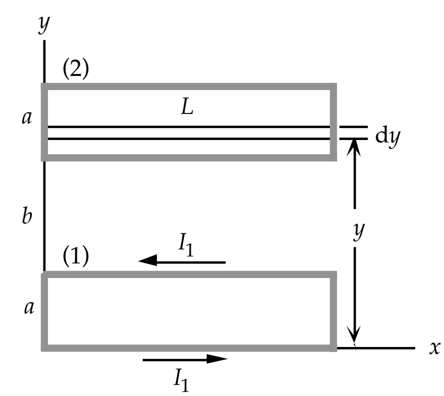
The mutual inductance is

$$M = \Phi_B/I = \boxed{(\mu_0 a/2\pi)\ \ln(1 + a/d)}.$$

23. We find the mutual inductance by finding the flux through the upper circuit (2) produced by the current in the lower circuit (1). Because $a << L$, and $b << L$, we can ignore the magnetic field from the short sides of circuit (1). The magnetic field from a long wire is

$$B = \mu_0 I/2\pi y.$$

The two fields are in opposite directions. To find the flux through circuit (2), we choose a strip dy a distance y from the bottom wire:



$$\Phi_{B21} = \iint \vec{B} \cdot d\vec{A} \int_{a+b}^{2a+b} \left[\frac{\mu_0 I_1 L}{2\pi y} - \frac{\mu_0 I_1 L}{2\pi(y-a)}\right] dy$$

$$= \frac{\mu_0 I_1 L}{2\pi} \ln\left[\frac{(2a+b)b}{(a+b)^2}\right].$$

Because $(a + b)^2 > (2a + b)b$, this flux is negative, that is, into the paper.

For the mutual inductance, we have

$$M = \frac{|\Phi_{B21}|}{I_1} = \frac{\mu_0 L}{2\pi} \ln\left[\frac{(a+b)^2}{(2a+b)b}\right].$$

24. For the energy per unit length, we have

$$U/\ell = \tfrac{1}{2} L I^2/\ell = \tfrac{1}{2}\mu_0 A n^2 I^2$$

$$= \tfrac{1}{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.75 \times 10^{-2}\ \text{m})^2(140)^2(0.45\ \text{A})^2 = \boxed{4.4 \times 10^{-7}\ \text{J/m}}.$$

25. We find the current in the inductor from

$$U_L = \tfrac{1}{2}LI^2;$$

$$0.1 \times 10^6\ \text{J} = \tfrac{1}{2}(16\ \text{H})I^2, \text{ which gives } I = 1.1 \times 10^2\ \text{A}.$$

The rate at which energy is lost to Joule heating is

$$P = I^2R = (1.1 \times 10^2\ \text{A})^2(0.1\ \Omega) = 1.2 \times 10^3\ \text{W} = \boxed{1.2\ \text{kW}}.$$

26. We find the work required from the change in stored energy:

$$W = \Delta U = \tfrac{1}{2}L(I_2^2 - I_1^2) = \tfrac{1}{2}(1.6 \times 10^{-3}\ \text{H})[(0.098\ \text{A})^2 - (0.088\ \text{A})^2] = \boxed{1.5 \times 10^{-6}\ \text{J}}.$$

27. For the two stored energies to be equal, we have

$$U_C = U_L;$$

$$\tfrac{1}{2}Q^2/C = \tfrac{1}{2}LI^2;$$

$$\tfrac{1}{2}(15\ \mu\text{C})^2/(0.02\ \mu\text{F}) = \tfrac{1}{2}(20\ \mu\text{H})I^2, \text{ which gives } I = \boxed{24\ \text{A}}.$$

28. The energy in the inductor is

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\mu_0 AN^2I^2/\ell$$

$$= \tfrac{1}{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(6.0 \times 10^{-4}\text{m}^2)(600)^2(0.100\ \text{A})^2/(0.12\ \text{m}) = \boxed{1.1 \times 10^{-5}\ \text{J}}.$$

$\boxed{29}$. The energy in the inductor is

$$U = \tfrac{1}{2}LI^2.$$

Because of the decreasing current, the power expended is

$$P = -\,dU/dt = -\,LI\,dI/dt = -\,L(I_0\,e^{-\alpha t})\,I_0(-\,\alpha)\,e^{-\alpha t} = LI_0^{\,2}\alpha\,e^{-2\alpha t}$$

$$= (2\ \text{mH})(4.0\ \text{A})^2(0.02\ \text{s}^{-1})e^{-2(0.02\ /\text{s})t} = \boxed{0.64\ e^{-(0.04/\text{s})t}\ \text{mW}}.$$

30. The voltage across the inductor is

$$V = L\,dI/dt.$$

Because V and L are constant, dI/dt is constant; the current changes linearly with time. The power supplied by the external source is the rate at which the stored energy in the inductor is changing:

$$P = dU/dt = d(\tfrac{1}{2}LI^2)/dt = LI\,dI/dt = IV.$$

For the average power, we have

$$P_{av} = I_{av}V = \tfrac{1}{2}(I_i + I_f)V, \text{ because the change is linear.}$$

(a) For a current change from 0.0 A to 0.10 A, we have

$$P_{av} = \tfrac{1}{2}(0 + (0.25\ \text{A})(6.0\ \text{V}) = \boxed{0.75\ \text{W}}.$$

(b) For a current change from 0.10 A to 0.20 A, we have

$$P_{av} = \tfrac{1}{2}(0.25\ \text{A} + 0.35\ \text{A})(6.0\text{V}) = \boxed{1.8\ \text{W}}.$$

(c) For a current change from 0.20 A to 0.30 A, we have

$$P_{av} = \tfrac{1}{2}(0.35\ \text{A} + 0.40\ \text{A})(6.0\ \text{V}) = \boxed{2.3\ \text{W}}.$$

31. (a) The stored energies are

$$U_L = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(1\ \text{H})(10\ \text{A})^2 = 50\ \text{J};$$

$$U_C = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}(It)^2/C = \tfrac{1}{2}I^2t^2/C = \tfrac{1}{2}(10\ \text{A})^2(1\ \text{s})^2/(1\ \text{F}) = 50\ \text{J}.$$

The two energies are $\boxed{\text{the same}}$.

(b) The stored energies are

$$U_L = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(1\ \text{H})(10^{-3}\ \text{A})^2 = 0.50\ \mu\text{J};$$

$$U_C = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}(It)^2/C = \tfrac{1}{2}I^2t^2/C = \tfrac{1}{2}(10^{-3}\ \text{A})^2(1\ \text{s})^2/(1\ \text{F}) = 0.50\ \mu\text{J}.$$

The two energies are the same.

32. (a) We assume that the wire is thin enough that only one layer of turns is necessary. The number of turns is

$N = L_{\text{wire}}/2\pi R.$

The inductance of the solenoid is

$L = \mu_0 A n^2 \ell = \mu_0 A (N/\ell)^2 \ell = \mu_0 A (L_{\text{wire}}/2\pi R \ell)^2 \ell$

$= (\mu_0/4\pi) A (L_{\text{wire}})^2/\pi R^2 \ell = (\mu_0/4\pi)(L_{\text{wire}})^2/\ell$

$= (10^{-7}\ \text{T}\cdot\text{m/A})(150\ \text{m})^2/(0.25\ \text{m}) = 9.0 \times 10^{-3}\ \text{H} = \boxed{9.0\ \text{mH}}.$

(b) The maximum stored energy is

$U_{\max} = \frac{1}{2}LI_{\max}{}^2 = \frac{1}{2}(9.0 \times 10^{-3}\ \text{H})(50 \times 10^{-3}\ \text{A})^2 = \boxed{1.1 \times 10^{-5}\ \text{J}}.$

33. (a) The energy in the solenoid is

$U = \frac{1}{2}LI^2$

$= \frac{1}{2}(8\ \text{H})(40\ \text{A})^2 = \boxed{6.4 \times 10^3\ \text{J}}.$

(b) We find the volume of helium evaporated from

$Q_v = L_v V;$

$6.4 \times 10^3\ \text{J} = (2.7 \times 10^3\ \text{J/L})V$, which gives $V = \boxed{2.4\ \text{L}}$.

34. The magnetic energy in the field is

$U = u_B V \simeq \frac{1}{2}(B^2/\mu_0)\frac{4}{3}\pi r^3$

$\simeq \frac{1}{2}[(10^{-10}\ \text{T})^2/(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})]\frac{4}{3}\pi[30.1(1.50 \times 10^{11}\ \text{m})]^3 \simeq \boxed{1.5 \times 10^{24}\ \text{J}}.$

35. The magnetic energy in the field is

$U = u_B V = \frac{1}{2}(B^2/\mu_0)(\frac{1}{4}\pi D^2)L$

$= \frac{1}{2}[(0.10\ \text{T})^2/(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})][\frac{1}{4}\pi(63 \times 10^{-2}\ \text{m})^2](0.21\ \text{m}) = \boxed{2.6 \times 10^2\ \text{J}}.$

36. The energy density of the magnetic field is

$u_B = \frac{1}{2}B^2/\mu = \frac{1}{2}(\mu n I)^2/\mu = \frac{1}{2}\mu n^2 I^2$

$= \frac{1}{2}(5500)(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(15 \times 10^2)^2(115 \times 10^{-3}\ \text{A})^2 = \boxed{103\ \text{J/m}^3}.$

37. The energy density of the magnetic field is

$u_B = \frac{1}{2}B^2/\mu_0 = \frac{1}{2}(\mu_0 I/2\pi r)^2/\mu_0 = \mu_0 I^2/8\pi^2 r^2$

$= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(20\ \text{A})^2/8\pi^2 r^2 = \boxed{(6.4 \times 10^{-6}\ \text{J/m})/r^2}.$

The energy density of the electric field of the capacitor is

$u_C = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (V/d)^2 = \frac{1}{2}\varepsilon_0 (Q/Cd)^2$

$= \frac{1}{2}(8.85 \times 10^{-12}\ \text{F/m})[(10^{-7}\ \text{C})/(6.3 \times 10^{-9}\ \text{F})(1.5 \times 10^{-3}\ \text{m})]^2$

$= 5.0 \times 10^{-4}\ \text{J/m}^3.$

For the energies to be equal, we have

$(6.4 \times 10^{-6}\ \text{J/m})/r^2 = 5.0 \times 10^{-4}\ \text{J/m}^3$, which gives

$r = 0.11\ \text{m} = \boxed{11\ \text{cm}}.$

38. (a) The magnetic field outside the wire is independent of the radius of the wire. The energy density of the magnetic field is

$u_B = \frac{1}{2}B^2/\mu_0 = \frac{1}{2}(\mu_0 I/2\pi r)^2/\mu_0 = \boxed{\mu_0 I^2/8\pi^2 r^2}.$

(b) Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell centered on the wire, with radius $r > a$, thickness dr, and length L. The energy per unit length is

$$\frac{U_B}{L} = \frac{1}{L}\int_a^R \frac{\mu_0 I^2}{8\pi^2 r^2} L 2\pi r\ dr = \frac{\mu_0 I^2}{4\pi}\int_a^R \frac{dr}{r} = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{R}{a}\right).$$

39. The energy stored in the inductor is the energy stored in the magnetic field:
$U_L = \frac{1}{2}LI^2 = \frac{1}{2}(B^2/\mu_0)A\ell$, which gives
$L = (B/I)^2A\ell/\mu_0 = (\mu_0 n)^2(b^2)(2\pi R)/\mu_0 = 2\pi\mu_0 n^2Rb^2$.

40. (a) The magnetic energy density inside the gap is
$u_B = \frac{1}{2}B^2/\mu_0$. So the total magnetic energy within the gap of volume $V = Ay$ is
$U_B = u_BV = \frac{1}{2}B^2Ay/\mu_0$. Suppose one pulls on the lower half of the iron ring with a force F equal in magnitude and opposite in direction to the magnetic force to increase the gap width by dy, then the work done by the force is
$dW = F\,dy = dU_B = \frac{1}{2}B^2A\,dy/\mu_0$. Thus
$F = dU_B/dy = \boxed{\frac{1}{2}B^2A/\mu_0}$.
(b) We now need to replace B with μB and μ_0 with μ. Thus F becomes μF, so it $\boxed{\text{increases}}$.

41. As the small cylinder falls into the hollow solenoid, the magnetic flux through the cross-section of the cylinder increases. By Faraday's law, an induced emf is established in the iron cylinder, and a current results. The magnetic force exerted by the magnetic field of the solenoid on the induced current in the cylinder is upward, opposing its fall. The magnitude of the force is proportional to the strength of the magnetic field, the cross-sectional area of the cylinder, as well as its descending speed. Once the cylinder completely enters the solenoid, the magnetic flux through it no longer changes, and the induced emf in it, along with the resulting magnetic force, diminishes to zero.

42. (a) From the cylindrical symmetry, we know that the magnetic field is circular. We apply Ampere's law to a circular path to find the magnetic field inside the wire:
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$;
$B2\pi r = \mu_0(I/\pi a^2)\pi r^2$, which gives $B = \mu_0 Ir/2\pi a^2$.
The energy density of the magnetic field is
$u_B = \frac{1}{2}B^2/\mu_0 = \boxed{\mu_0 I^2r^2/8\pi^2a^4}$.
(b) Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell centered on the wire, with radius $r < a$, thickness dr, and length L. The energy per unit length is

$$\frac{U_B}{L} = \frac{1}{L}\int_0^a \frac{\mu_0 I^2r^2}{8\pi^2a^4}L2\pi r\,dr = \frac{\mu_0 I^2}{4\pi a^4}\int_0^a r^3\,dr = \frac{\mu_0 I^2}{16\pi}.$$

43. We assume that the current in the inner wire is uniform over its area. For the magnetic energy per unit length within the inner wire, we use the result from Problem 42:
$U_B/L = \mu_0 I^2/16\pi = (\mu_0/4\pi)I^2/4$
$= (10^{-7}\ \text{T}\cdot\text{m/A})(0.25\ \text{A})^2/4 = \boxed{1.6\times10^{-9}\ \text{J/m}}$ inside the inner wire.

44. For an RL circuit, we have
$I = I_{\text{max}}(1 - e^{-Rt/L})$.
To be within 18% of the steady state value, I_{max}, we have
$e^{-Rt/L} = 0.18$, or $Rt/L = 1.7$, which gives
$R(5\times10^{-4}\ \text{s})/L = 1.7$, or $R \simeq 3.4\times10^3\ L$.
For 0.01 H < L < 0.1 H, we get $\boxed{34\ \Omega < R < 3.4\times10^2\ \Omega}$.

45. For the dimensions of the time constant, we have
$[L/R] = [L]\,[R]^{-1} = [ML^2Q^{-2}]\,[Q^2TM^{-1}L^{-2}] = [T]$.

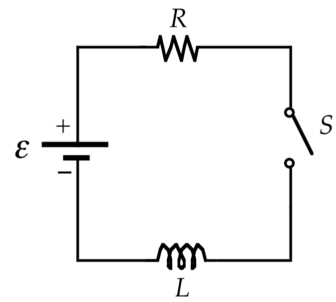
46. The current in the circuit is

$I = (\mathcal{E}/R)(1 - e^{-Rt/L})$.

The time constant of the system is

$t_c = L/R = (2.5 \times 10^{-3}\text{ H})/(3.3 \times 10^3\ \Omega) = 7.58 \times 10^{-7}$ s.

We find the charge that has moved through any element in the circuit by integrating:



$$Q = \int_0^T I\,dt = \frac{\mathcal{E}}{R}\int_0^T \left(1 - e^{-Rt/L}\right) dt$$

$$= \frac{\mathcal{E}}{R}\left[T - \frac{L}{R}\left(1 - e^{-RT/L}\right)\right] = \frac{\mathcal{E}}{R}\left[T - t_c\left(1 - e^{-T/t_c}\right)\right].$$

(a) For a time of 1 μs, we have

$$Q = \frac{6\text{ V}}{3.3 \times 10^3\ \Omega}\left\{(1 \times 10^{-6}\text{ s}) - (7.58 \times 10^{-7}\text{ s})\left[1 - e^{-(1 \times 10^{-6}\text{ s})/(7.58 \times 10^{-7}\text{ s})}\right]\right\} = 8.1 \times 10^{-10}\text{ C.}$$

(b) For a time of 1 ms, we have

$$Q = \frac{6\text{ V}}{3.3 \times 10^3\ \Omega}\left\{(1 \times 10^{-3}\text{ s}) - (7.58 \times 10^{-7}\text{ s})\left[1 - e^{-(1 \times 10^{-3}\text{ s})/(7.58 \times 10^{-7}\text{ s})}\right]\right\} = 1.8 \times 10^{-6}\text{ C.}$$

(c) For a time of 1 s, we have

$$Q = \frac{6\text{ V}}{3.3 \times 10^3\ \Omega}\left\{(1\text{ s}) - (7.58 \times 10^{-7}\text{ s})\left[1 - e^{-(1\text{ s})/(7.58 \times 10^{-7}\text{ s})}\right]\right\} = 1.8 \times 10^{-3}\text{ C.}$$

47. From the expression for the current in the circuit,

$I = (\mathcal{E}/R)(1 - e^{-Rt/L})$, we get

$dI/dt = (\mathcal{E}/R)[-(-R/L)]e^{-Rt/L}] = +(\mathcal{E}/L)\,e^{-Rt/L}$.

When we use these expressions in the left–hand side of Eq. (32–18), we get

$\mathcal{E} - IR - (L\,dI/dt) = \mathcal{E} - (\mathcal{E}/R)R(1 - e^{-Rt/L}) - L(\mathcal{E}/L)e^{-Rt/L}$

$= \mathcal{E} - \mathcal{E} + \mathcal{E}(e^{-Rt/L}) - \mathcal{E}(e^{-Rt/L}) = 0$,

so Eq. (32–18) is satisfied by the expression for the current.

48. Before the switch is open there is a stable current in the inductor. After the switch is opened at $t = 0$ we have an RL circuit, with $R = 15\ \Omega + 20\ \Omega = 35\ \Omega$ and $L = 0.5$ H. The current I as a function of time is

$I = I_0\,e^{-Rt/L}$, and so

$t = (L/R)\ln(I_0/I)$

$= [(0.5\text{ H})/(35\ \Omega)]\ln(I_0/0.25I_0) = [(0.5\text{ H})/(35\ \Omega)]\ln 4.0 = 0.020\text{ s} = \boxed{20\text{ ms}}$.

Since $U_L = \frac{1}{2}LI^2$ is proportional to I^2, it decrease to 25% of its initial value as I^2 drops to $0.25\,I_0^2$, i.e., as I drops to $(0.25)^{1/2}\,I_0 = 0.50\,I_0$. The time it takes is

$t = (L/R)\ln(I_0/I) = (0.5\text{ H}/35\ \Omega)\ln(I_0/0.50I_0) = \boxed{10\text{ ms}}$.

$\boxed{49}$. The current in an RL circuit a time t after the power is turned on is

$I = (\mathcal{E}/R)(1 - e^{-Rt/L})$. Since 10 s >> 10^{-4} s we can practically think of $t = 10$ s as infinity, at which time $e^{-Lt/R}$ approaches zero and the current is

$I = \mathcal{E}/R = 0.48$ A; so $R = \mathcal{E}/I = 12\text{ V}/\ 0.48\text{ A} = \boxed{25\ \Omega}$.

Now consider $t = 1.2 \times 10^{-4}$ s, when $I = 0.2$ A. We have $e^{-Rt/L} = 1 - IR/\mathcal{E}$, or

$L = -Rt/\ln(1 - \mathcal{E}/IR)$

$= -(25\ \Omega)(1.2 \times 10^{-4}\text{ s})\ /\ln[1 - (0.2\text{ A})(25\ \Omega)/12\text{ V}] = 0.0056\text{ H} = \boxed{5.6\text{ mH}}$.

50. If we assume that the current decays exponentially, we write

$I = Ae^{-\alpha t}$, and $dI/dt = -A\alpha e^{-\alpha t}$.

When we put these in the circuit equation, we have

$L\, dI/dt + RI = 0$;

$-LA\alpha e^{-\alpha t} + RAe^{-\alpha t} = 0$, which gives $\alpha = R/L$.

When the short occurs at $t = 0$, we have

$I_0 = A = V/R$, so the result is

$I = \boxed{(V/R)e^{-Rt/L}}$.

51. Because the current varies, we find the energy dissipated by integrating:

$$W_R = \int_0^\infty I^2 R\, dt = \left(\frac{V}{R}\right)^2 R \int_0^\infty e^{-2Rt/L}\, dt = \left(\frac{V^2}{R}\right)\left(\frac{-L}{2R}\right) e^{-2Rt/L}\Big|_0^\infty = \frac{V^2 L}{2R^2}.$$

For the initial energy stored in the inductor, we have

$$U_0 = \tfrac{1}{2} L I_0^2 = \tfrac{1}{2} L \left(\frac{V}{R}\right)^2 = W_R,$$

which is the energy dissipated in the resistance.

52. For the resonant frequencies of the two systems, we have

$\omega_0 = 1/(L_0C_0)^{1/2}$ and $\omega = 1/(LC)^{1/2}$.

When we form the ratio, we get

$\omega/\omega_0 = (L_0C_0/LC)^{1/2} = [(1/12)(1/12)]^{-1/2} = (144)^{1/2} = 12$, which gives $\omega = \boxed{12\omega_0}$.

53. We find the required capacitance from

$\omega = 2\pi f = (1/LC)^{1/2}$;

$2\pi(20 \text{ Hz}) = [1/(40 \times 10^{-3} \text{ H})C]^{1/2}$, which gives $C = 1.6 \times 10^{-3}$ F $= \boxed{1.6 \text{ mF}}$.

54. We estimate the inductance from

$L = \mu_0 A \ell n^2 = \mu_0 \pi r^2 N^2/\ell = \mu_0 \pi r^2 (1)^2/r = \mu_0 \pi r$.

We estimate the capacitance from

$C = \varepsilon_0 A/d = \varepsilon_0 \pi r^2/r = \varepsilon_0 \pi r$.

We find the estimated frequency from

$f = 1/2\pi(LC)^{1/2} = 1/2\pi[(\mu_0\pi r)(\varepsilon_0\pi r)]^{1/2} = 1/2\pi^2 r(\varepsilon_0\mu_0)^{1/2}$

$= 1/2\pi^2(10 \times 10^{-6} \text{ m})[(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})]^{1/2} = \boxed{1.5 \times 10^{12} \text{ Hz}}$.

$\boxed{55}$. From the problem statement we know that

$1/(LC)^{1/2} = \omega_0$ and $U = \tfrac{1}{2}CV_{\max}^2$. Solve for C from the last equation:

$C = 2U/V_{\max}^2$

$= 2(0.30 \text{ mJ})/(20.0 \text{ V})^2 = 1.5 \times 10^{-3} \text{ mF} = \boxed{1.5\ \mu\text{F}}$; and so

$L = 1/C\omega_0^2 = 1/[(1.5 \times 10^{-6} \text{ F})(4.32 \times 10^4 \text{ rad/s})^2] = 3.6 \times 10^{-4} \text{ H} = \boxed{0.36 \text{ mH}}$.

56. (a) At $t = 0$ there is no current, so the energy is entirely stored on the capacitor:

$U = \boxed{\tfrac{1}{2}Q(0)^2/C}$.

(b) Since there is no charge on the capacitor at $t = 0$ the energy is entirely stored on the inductor:

$U = \boxed{\tfrac{1}{2}L\, I(0)^2}$.

(c) If we start with all electric or all magnetic energy at $t = 0$, such as in cases (a) and (b), then we need to wait till the electric (or magnetic) energy to decrease to half of its initial value. For example, in case (a), since U_C is proportional to Q^2, Q must drop to $1/\sqrt{2}$ of $Q(0)$:

$Q(t) = Q(0) \cos \omega t = Q(0)/\sqrt{2}$, or $\cos \omega t = 1/\sqrt{2}$;

$t = \pi/4\omega = \pi/[4/(LC)^{1/2}] = \pi(LC)^{1/2}/4 \approx \boxed{0.785(LC)^{1/2}}$.

(d) Compare the expression of energies in the mechanical and LC oscillation cases:

Energy = $\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$ (mechanical oscillation); Energy = $\tfrac{1}{2}LI^2 + \tfrac{1}{2}CV^2$ (LC oscillation).

It is clear that L corresponds to m and I to v. Thus the electrical analog of momentum, mv, is $\boxed{LI}$.

57. The scaling of any length is $a_2 = 10a_1$. For the resistance we have
$R_2 = \rho\ell_2/A_2 = \rho(10\ell_1)/100A_1 = R_1/10$.
For the inductance we have
$L_2 = \mu A_2N^2/\ell_2 = \mu(100A_1)N^2/10\ell_1 = 10L_1$.
For the capacitance we have
$C_2 = \varepsilon A_2/d_2 = \varepsilon(100A_1)/10d_1 = 10C_1$.
(a) For the undamped frequency we have
$\omega_2 = 1/(L_2C_2)^{1/2} = 1/(10L_1\,10C_1)^{1/2} = \boxed{\omega_1/10}$.
(b) For the damped factor we have
$\alpha_2 = R_2/2L_2 = (R_1/10)/2(10L_1) = \boxed{\alpha_1/100}$.
(c) For the damped frequency we have
$\boxed{\omega_2'^2 = \omega_2^2 - \alpha_2^2 = (\omega_1^2/100) - (\alpha_1/100)^2}$.
There is no direct proportionality. Damping becomes less important for the larger circuit.

58. The general expression for the charge on the capacitor is
$Q = Q_0 \cos(\omega t + \phi)$, so the current is
$I = dQ/dt = -Q_0\omega \sin(\omega t + \phi)$.
At $t = 0$, we have
$Q = q = Q_0 \cos\phi$, and $I = 0 = -Q_0\omega \sin\phi$, which gives $\phi = 0$, and $Q_0 = q$.
The current is
$I = -q\omega \sin(\omega t)$, so the maximum magnitude of the current is
$I_{\max} = q\omega = \boxed{q/(LC)^{1/2}}$.
This maximum magnitude will occur when $\sin(\omega t) = \pm 1$, or $\omega t = \pi/2, 3\pi/2, 5\pi/2, ...$
The times when the maximum occurs are
$t = (2n-1)\pi/2\omega = \boxed{\tfrac{1}{2}(2n-1)\pi(LC)^{1/2},\ n = 1, 2, 3, ...}$.

59. The damping factor is
$\alpha = R/2L = (0.883\ \Omega)/2(1.75\ \text{H}) = 0.252\ \text{s}^{-1}$.
We find the angular frequency from
$\omega'^2 = (1/LC) - (R^2/4L^2) = (1/LC) - \alpha^2$
$= [1/(1.75\ \text{H})(133 \times 10^{-12}\ \text{F})] - (0.252\ \text{s}^{-1})^2$, which gives $\omega' = \boxed{6.6 \times 10^4\ \text{rad/s.}}$
Because the damping factor is small, this is close to the undamped frequency.

60. (a) The damping factor is
$\alpha = R/2L = (0.085\ \Omega)/2(0.60 \times 10^{-3}\ \text{H}) = \boxed{71\ \text{s}^{-1}}$.
We find the angular frequency from
$\omega'^2 = (1/LC) - (R^2/4L^2) = (1/LC) - \alpha^2$
$= [1/(0.60\times 10^{-3}\ \text{H})(55 \times 10^{-6}\ \text{F})] - (71\ \text{s}^{-1})^2$, which gives $\omega' = \boxed{5.5 \times 10^3\ \text{rad/s}}$.
(b) For critical damping, we have
$R_c^2 = 4L/C = 4[(0.60 \times 10^{-3}\ \text{H})/(55 \times 10^{-6}\ \text{C})]$, which gives $R_c = \boxed{6.6\ \Omega}$.

61. For critical damping, we have
$R_c = 2(L/C)^{1/2}$, and $\alpha = R_c/2L = 1/(LC)^{1/2}$.
The charge on the capacitor is
$Q = Q_0e^{-\alpha t}\cos(\omega' t + \phi) = Q_0e^{-\alpha t}\cos(\omega' t)$, so $Q = Q_0$ when $t = 0$.
The current is
$I = dQ/dt = Q_0e^{-\alpha t}[-\omega' \sin(\omega' t)] + Q_0(-\alpha)e^{-\alpha t}\cos(\omega' t)$, so
$I_0 = -Q_0\alpha$ when $t = 0$.
For the instantaneous power consumption in the resistor, we have
$P = I^2R_c = (-Q_0\alpha e^{-\alpha t})^2R_c = Q_0^2(1/LC)e^{-2\alpha t}[2(L/C)^{1/2}] = \quad 2Q_0^2/(LC^3)^{1/2}e^{-2\alpha t}$.

62. For a critically-damped circuit, the current is

$I = I_0 e^{-\alpha t}$;

$0.85 I_0 = I_0 e^{-\alpha(0.0080\text{ s})}$, which gives $\alpha = 20\text{ s}^{-1}$.

We find the resistance from

$\alpha = R_c/2L$;

$20\text{ s}^{-1} = R_c/2(68 \times 10^{-3}\text{ H})$, which gives $R_c = \boxed{2.7\ \Omega}$.

63. From the expression for the charge,

$Q = Q_0 e^{-\alpha t}\cos(\omega' t + \phi)$, we get

$I = \mathrm{d}Q/\mathrm{d}t = Q_0 e^{-\alpha t}[-\omega' \sin(\omega' t + \phi)] + Q_0(-\alpha)e^{-\alpha t}\cos(\omega' t + \phi)$

$= Q_0 e^{-\alpha t}[-\omega' \sin(\omega' t + \phi) - \alpha \cos(\omega' t + \phi)]$;

$\mathrm{d}I/\mathrm{d}t = Q_0 e^{-\alpha t}[-\omega'^2 \cos(\omega' t + \phi) - \alpha(-\omega')\sin(\omega' t + \phi)] +$

$Q_0(-\alpha)e^{-\alpha t}[-\omega' \sin(\omega' t + \phi) - \alpha\cos(\omega' t + \phi)]$

$= Q_0 e^{-\alpha t}[(\alpha^2 - \omega'^2)\cos(\omega' t + \phi) + 2\alpha\omega' \sin(\omega' t + \phi)]$.

For the three terms on the left-hand side of Eq. (32–27), we have

$-L\,\mathrm{d}I/\mathrm{d}t = +Q_0 L e^{-\alpha t}[(\omega'^2 - \alpha^2)\cos(\omega' t + \phi) + 2\alpha\omega' \sin(\omega' t + \phi)]$;

$-IR = Q_0 R e^{-\alpha t}[+\omega' \sin(\omega' t + \phi) + \alpha\cos(\omega' t + \phi)]$;

$-Q/C = -(Q_0/C)e^{-\alpha t}\cos(\omega' t + \phi)$.

When we add the three terms, we get

$(-L\,\mathrm{d}I/\mathrm{d}t) - IR - (Q/C) =$

$Q_0 e^{-\alpha t}\{[(\omega'^2 - \alpha^2)L + R\alpha - 1/C]\cos(\omega' t + \phi) + (-2\alpha\omega' L + R\omega')\sin(\omega' t + \phi)\}$.

We use Eqs. (32–31) and (32–32) in the coefficients of the trigonometric functions:

cosine term: $(\omega'^2 - \alpha^2)L + R\alpha - (1/C) = [(1/LC) - \alpha^2 - \alpha^2]L + R\alpha - (1/C)$

$= \alpha(-2\alpha L + R) = \alpha(-R + R) = 0$;

sine term: $-2\alpha\omega' L + R\omega' = \omega'(-2\alpha L + R) = 0$.

Thus the three terms add to zero, which is Eq. (32–27).

64. For the trial solution

$Q = Q_1 e^{-\alpha_1 t} + Q_2 e^{-\alpha_2 t}$, we get

$I = \dfrac{\mathrm{d}Q}{\mathrm{d}t} = -Q_1\alpha_1 e^{-\alpha_1 t} - Q_2\alpha_2 e^{-\alpha_2 t}$;

$\dfrac{\mathrm{d}I}{\mathrm{d}t} = +Q_1\alpha_1^2 e^{-\alpha_1 t} + Q_2\alpha_2^2 e^{-\alpha_2 t}$.

When we substitute these in Eq. (32–27), we get

$-L\left(Q_1\alpha_1^2 e^{-\alpha_1 t} + Q_2\alpha_2^2 e^{-\alpha_2 t}\right) + R\left(Q_1\alpha_1 e^{-\alpha_1 t} + Q_2\alpha_2 e^{-\alpha_2 t}\right) - (1/C)\left(Q_1 e^{-\alpha_1 t} + Q_2 e^{-\alpha_2 t}\right) = 0$;

$Q_1\left[-L\alpha_1^2 + R\alpha_1 - (1/C)\right]e^{-\alpha_1 t} + Q_2\left[-L\alpha_2^2 + R\alpha_2 - (1/C)\right]e^{-\alpha_2 t} = 0$.

For arbitrary t, the coefficients of the exponential terms must be zero. α_1 and α_2 are the solutions to the same quadratic equation:

$L\alpha_1{}^2 - R\alpha_1 + (1/C) = 0$, which has the solutions $(R/2L) \pm [(R/2L)^2 - (1/LC)]^{1/2}$;

$\boxed{\alpha_1 = \alpha + (-\omega'^2)^{1/2}, \quad \text{and} \quad \alpha_2 = \alpha - (-\omega'^2)^{1/2}}$.

Because $\omega'^2 < 0$, α_1 and α_2 are real.

65. The angular frequency of the damped circuit is

$\omega' = (1/LC - R^2/4L^2)^{1/2} = (\omega^2 - \alpha^2)^{1/2} = \omega(1 - \alpha^2/\omega^2)^{1/2}$.

If $\alpha << \omega$, we use the approximation $(1 - x)^{1/2} \simeq 1 - \frac{1}{2}x$:

$\omega' \simeq \omega[1 - \frac{1}{2}(\alpha^2/\omega^2)] \simeq \omega[1 - \frac{1}{2}(R^2/4L^2)(LC)] \simeq \omega - R^2(C/L)^{1/2}/8L$.

The period of the undamped circuit is $T = 2\pi/\omega$. For the slightly damped case, we have

$T' = 2\pi/\omega' = 2\pi/(\omega^2 - \alpha^2)^{1/2} = (2\pi/\omega)(1 - \alpha^2/\omega^2)^{-1/2}$

$\simeq T[1 + \frac{1}{2}(\alpha^2/\omega^2)] = T[1 + \frac{1}{2}(R^2/4L^2)(LC)] = \boxed{T + \frac{1}{4}\pi R^2(C^3/L)^{1/2}}$.

66. From the given expression for the charge on the capacitor,

$Q = Q_0 \cos(\omega t + \phi)$, we find the current:

$I = dQ/dt = -Q_0\omega \sin(\omega t + \phi)$.

The energy of the circuit is

$U = U_C + U_L = \frac{1}{2}(Q^2/C) + \frac{1}{2}LI^2$

$= \frac{1}{2}(Q_0{}^2/C)\cos^2(\omega t + \phi) + \frac{1}{2}LQ_0{}^2\omega^2\sin^2(\omega t + \phi)$

$= \frac{1}{2}(Q_0{}^2/C)[\cos^2(\omega t + \phi) + LC\omega^2\sin^2(\omega t + \phi)]$.

The angular frequency of the circuit is $\omega = (1/LC)^{1/2}$, so we have

$U = \frac{1}{2}(Q_0{}^2/C)[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)] = \boxed{Q_0{}^2/2C}$, which is a constant.

$\boxed{67}$. (a) For the charge on the capacitor, we can write

$Q = Q_0 \cos(\omega t)$, where Q_0 is the initial charge at $t = 0$.

The current in the circuit is

$I = dQ/dt = -Q_0\omega \sin(\omega t)$.

The maximum current is

$I_{\max} = Q_0\omega = Q_0/(LC)^{1/2} = (30 \times 10^{-9}\text{ C})/[(2 \times 10^{-5}\text{ H})(20 \times 10^{-9}\text{ F})]^{1/2} = 4.7 \times 10^{-2}\text{ A} = \boxed{47\text{ mA}}$.

(b) The maximum energy stored in the inductor is

$U_{L\max} = \frac{1}{2}LI_{\max}{}^2 = \frac{1}{2}(2 \times 10^{-5}\text{ H})(4.7 \times 10^{-2}\text{ A})^2 = \boxed{2.2 \times 10^{-8}\text{ J}}$.

(c) The ratio of the two maximum energies is

$U_{L\max}/U_{C\max} = \frac{1}{2}LI_{\max}{}^2/(\frac{1}{2}Q_0{}^2/C) = L(Q_0\omega)^2/(Q_0{}^2/C) = LC\omega^2 = \boxed{1}$.

Note that this must be so from the conservation of energy.

68. We find the maximum charge on the capacitor from the maximum energy stored in the capacitor:

$U_{C\max} = \frac{1}{2}Q_{\max}{}^2/C$;

$3 \times 10^{-4}\text{ J} = \frac{1}{2}Q_{\max}{}^2/(120 \times 10^{-6}\text{ F})$, which gives $Q_{\max} = \boxed{2.7 \times 10^{-4}\text{ C}}$.

We find the maximum current in the circuit from the maximum energy stored in the inductor:

$U_{L\max} = \frac{1}{2}LI_{\max}{}^2$;

$3 \times 10^{-4}\text{ J} = \frac{1}{2}(15 \times 10^{-3}\text{ H})I_{\max}{}^2$, which gives $I_{\max} = \boxed{0.2\text{ A}}$.

Note that this is consistent with $I_{\max} = Q_{\max}\omega$.

The minimum values, which occur at different times, are

$Q_{\min} = \boxed{0}$ and $I_{\min} = \boxed{0}$.

69. (a) For the angular frequency of an *RLC* circuit, we have

$\omega'^2 = \omega^2 - \alpha^2$, where $\alpha = R/2L$.

If the resistance is very small, we have $\omega' \simeq \omega$.

The charge on the capacitor is

$Q = Q_0 e^{-\alpha t}\cos(\omega' t) \simeq Q_0 e^{-\alpha t}\cos(\omega t)$.

At $t = 0$, $Q = Q_0$, and at the end of each oscillation, $\cos(\omega t) = 1$. So after 100 oscillations, we have

$5\ \mu\text{C} = (30\ \mu\text{C})e^{-\alpha t}$, with $t = 100(2\pi/\omega)$, which gives

$\alpha t = 1.79 = \alpha(100)2\pi/\omega = \alpha 200\pi(LC)^{1/2}$;

$\alpha 200\pi[(1.5 \times 10^{-3}\text{ H})(3 \times 10^{-3}\text{ F})]^{1/2} = 1.79$, which gives $\alpha = 1.34\text{ s}^{-1}$.

We find the resistance from

$\alpha = R/2L$;

$1.34\text{ s}^{-1} = R/2(1.5 \times 10^{-3}\text{ H})$, which gives $R = \boxed{4.0 \times 10^{-3}\ \Omega.}$

(b) We find the energies of the circuit from the maximum energies of the capacitor:

$U_0 = \frac{1}{2}Q_0{}^2/C = \frac{1}{2}(30 \times 10^{-6}\text{ C})^2/(3 \times 10^{-3}\text{ F}) = \boxed{1.5 \times 10^{-7}\text{ J}}$ at $t = 0$;

$U = \frac{1}{2}Q^2/C = \frac{1}{2}(5 \times 10^{-6}\text{ C})^2/(3 \times 10^{-3}\text{ F}) = \boxed{4.2 \times 10^{-9}\text{ J}}$ at $t = 100$ oscillations.

(c) The two energies are not the same because energy has been lost from $\boxed{\text{Joule heating}}$.

70. The equation for the RLC circuit is

$$-L\,(dI/dt) - IR - Q/C = 0.$$

We differentiate the expression for the stored energy to find its rate of change:

$$E = \tfrac{1}{2}LI^2 + \tfrac{1}{2}Q^2/C;$$

$$dE/dt = LI(dI/dt) + (Q/C)(dQ/dt) = I[L(dI/dt) + Q/C].$$

When we use the circuit equation, we get

$dE/dt = I(-IR) = -I^2R$, which is the power loss in the resistor.

71. From the definition of inductance, $L = \Phi_B/I$, we have

$I = \Phi_B/L$, and

$$dI/dt = (1/L)\,d\Phi_B/dt = (1/L)(-\mathcal{E}) = -\mathcal{E}/L.$$

If we assume the situation where the impressed voltage causes an increase in current, the induced emf opposes the voltage: $\mathcal{E} = -V$, so we have

$$dI = +(1/L)V\,dt.$$

Because V changes with time, the total current passing through the inductor is

$$I = (1/L)\int V\,dt.$$

72. We assume that $I = 0$ at $t = 0$. For the segment of the square wave between $t = 0$ and $t = 0.1$ s, we have

$$I = (1/L)\int V\,dt = (V/L)\int dt = (V/L)t = [(1\text{ V})/(0.005\text{ H})]t = (200\text{ A/s})t.$$

The current increases linearly to

$$I_{max} = (200\text{ A/s})(0.1\text{ s}) = \boxed{20\text{ A}}.$$

For the next segment, where the voltage is negative, the current will decrease to zero. Then the cycle will repeat.

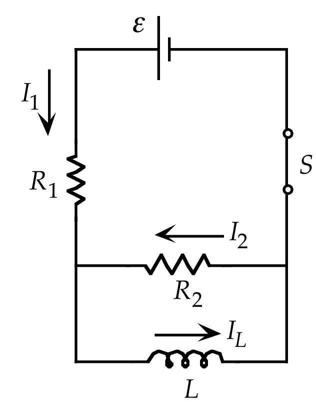
73. (a) When the switch has been closed for a long time, the currents will be constant. Because there is no change incurrent through the inductor, there will be no emf in the inductor and thus no potential difference across it.
Because the resistor R_2 is in parallel with the inductor, the potential difference across it must also be zero; the current through the 24-Ω resistor is

$$I_2 = 0.$$

For the outside loop, we have

$$I_1 = I_\varepsilon = \mathcal{E}/R_1 = (12\text{ V})/(5\times10^3\ \Omega) = 2.4\times10^{-3}\text{ A, so}$$

$$I_1 = I_\varepsilon = I_L = \boxed{2.4\text{ mA}}.$$



(b) When the switch is opened, there is no current through the battery and R_1 , so we have

$$I_L = I_0e^{-R_2t/L};$$

$\tfrac{1}{2}I_0 = I_0e^{-(24\,\Omega)(8\times10^{-6}\text{ s})/L}$, which gives

$$L = 2.8\times10^{-4}\text{ H} = \boxed{0.28\text{ mH}}.$$

(c) Because the switch is open, there will be no current through R_1 and the battery:

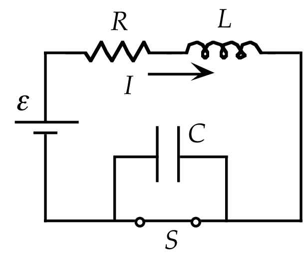
$$I_1 = I_\varepsilon = \boxed{0}.$$

For the inductor, we have

$$I_L = I_0e^{-R_2t/L};$$

$= \left(2.4\times10^{-3}\text{ A}\right)e^{-(24\,\Omega)(12\times10^{-6}\text{ s})/(2.8\times10^{-4}\text{ H})}$, which gives

$$I_L = I_2 = 8.6\times10^{-4}\text{ A} = \boxed{0.86\text{ mA}}.$$



74. If we assume that very little power is dissipated while the capacitor is initially charging, most of the energy stored in the magnetic field of the coil will be stored in the electric field of the capacitor. The initial current is

$I_0 = \mathcal{E}/R = (12\text{ V})/(5\ \Omega) = 2.4\text{ A}.$

We equate the two energies, with the maximum voltage allowed:

$\tfrac{1}{2}LI^2 = \tfrac{1}{2}CV^2;$

$(0.1\text{ H})(2.4\text{ A})^2 = C(200\text{ V})^2$, which gives

$C = 1.44 \times 10^{-5}\text{ F} = \boxed{14.4\ \mu\text{F}}.$

This capacitor will suffice because, if there is dissipation, the voltage will be less.

75. From the cylindrical symmetry, we know that the magnetic field is circular. We have three regions to consider.

Region I, inside the inner wire, $r < r_1$:

We apply Ampere's law to a circular path to find the magnetic field inside the wire:

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$

$B_\text{I} 2\pi r = \mu_0 (I/\pi r_1^2)\pi r^2$, which gives $B_\text{I} = \mu_0 I r / 2\pi r_1^2$.

The energy density of the magnetic field is

$u_{B\text{I}} = \tfrac{1}{2}B_\text{I}^2/\mu_0 = \mu_0 I^2 r^2 / 8\pi^2 r_1^4.$

Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell centered on the wire, with radius $r < r_1$, thickness dr, and length L. The energy per unit length is

$$\frac{U_{B\text{I}}}{L} = \frac{1}{L}\int_0^a \frac{\mu_0 I^2 r^2}{8\pi^2 a^4} L 2\pi r\, dr = \frac{\mu_0 I^2}{4\pi a^4}\int_a^R r^3\, dr = \frac{\mu_0 I^2}{16\pi}.$$

Region II, between the two wires, $r_1 < r < r_2$:

We apply Ampere's law to a circular path to find the magnetic field inside the wire:

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$

$B_\text{II} 2\pi r = \mu_0 I$, which gives $B_\text{II} = \mu_0 I / 2\pi r$.

The energy density of the magnetic field is

$u_{B\text{II}} = \tfrac{1}{2}B_\text{II}^2/\mu_0 = \mu_0 I^2 / 8\pi^2 r^2.$

Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell centered on the wire, with radius $r_1 < r < r_2$, thickness dr, and length L. The energy per unit length is

$$\frac{U_{B\text{II}}}{L} = \frac{1}{L}\int_{r_1}^{r_2} \frac{\mu_0 I^2}{8\pi^2 r^2} L\, 2\pi r\, dr = \frac{\mu_0 I^2}{4\pi}\int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{r_2}{r_1}\right).$$

Region III, outside the outer wire, $r_2 < r$:

We apply Ampere's law to a circular path to find the magnetic field outside the wire:

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$

$B_\text{III} 2\pi r = \mu_0 (I - I)$, which gives $B_\text{III} = 0$.

There is no magnetic energy outside the outer wire.

The total magnetic energy per unit length is

$U_{B\text{total}}/L = U_{B\text{I}}/L + U_{B\text{II}}/L$

$= \mu_0 I^2/16\pi + (\mu_0 I^2/4\pi)\ln(r_2/r_1)/L$

$= \boxed{(\mu_0 I^2/4\pi)[1/4 + \ln(r_2/r_1)].}$

76. The self-inductance of a solenoid is

$$L = \mu A n^2 \ell, \text{ where } \mu = (1 + \chi_m)\mu_0.$$

For the two temperatures, the only change is in the permeability, so we have

$$L_{20}/L_{300} = \mu_{20}/\mu_{300} = (1 + \chi_{20})/(1 + \chi_{300}).$$

The fractional change is

$$\begin{aligned}(L_{20} - L_{300})/L_{300} &= [(1 + \chi_{20}) - (1 + \chi_{300})]/(1 + \chi_{300}) = (\chi_{20} - \chi_{300})/(1 + \chi_{300}) \\ &= [(2.4 \times 10^{-4}) - (1.2 \times 10^{-4})]/[1 + (1.2 \times 10^{-4})] \\ &= \boxed{1.2 \times 10^{-4}}.\end{aligned}$$

77. (a) For the dimensions of E_0/B_0 , we have

$$[E/B_0] = [MLQ^{-1}T^{-2}]/[MQ^{-1}T^{-1}] = [LT^{-1}],$$ which are the dimensions of a velocity.

(b) Both fields will vary sinusoidally, so we have

$$\langle B^2\rangle = B_0^2\langle \sin^2\theta\rangle = \tfrac{1}{2}B_0^2, \text{ and } \langle E^2\rangle = E_0^2\langle \sin^2\theta\rangle = \tfrac{1}{2}E_0^2,$$

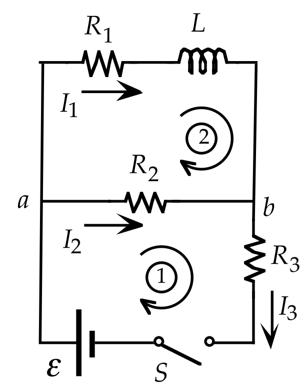
which gives the energy densties

$$u_B = \tfrac{1}{2}\langle B^2\rangle/\mu_0 = \tfrac{1}{2}(\tfrac{1}{2}B_0^2)/\mu_0 = \tfrac{1}{4}B_0^2/\mu_0;\ u_E = \tfrac{1}{2}\varepsilon_0\langle E^2\rangle = \tfrac{1}{2}\varepsilon_0(\tfrac{1}{2}E_0^2) = \tfrac{1}{4}\varepsilon_0 E_0^2.$$

If these are equal, we have

$$\tfrac{1}{4}\varepsilon_0 E_0^2 = \tfrac{1}{4}B_0^2/\mu_0, \text{ or } E_0/B_0 = (1/\mu_0\varepsilon_0)^{1/2} = \boxed{c}.$$

78. Immediately after the switch is closed, the induced emf in the inductor is maximum while the current in the inductor is zero, so we have



$$I_1 = \boxed{0}.$$

For loop 1, we have

$$I_2 = I_3 = \mathcal{E}/R_2 + R_3) = (1.5\text{ V})/(6\text{ k}\Omega + 2\text{ k}\Omega) = \boxed{0.19\text{ mA}}.$$

After a long time, the currents will be constant, and there will be no induced emf in the inductor. For the junction at point a, we have

$$I_3 = I_1 + I_2.$$

For loop 1, we have

$$\mathcal{E} - I_2R_2 - I_3R_3 = 0;\quad 1.5\text{ V} - I_2(6\text{ k}\Omega) - I_3(2\text{ k}\Omega) = 0.$$

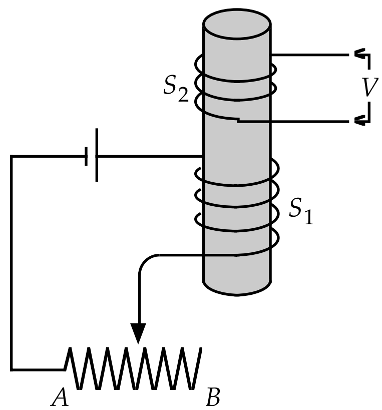
For loop 2, we have

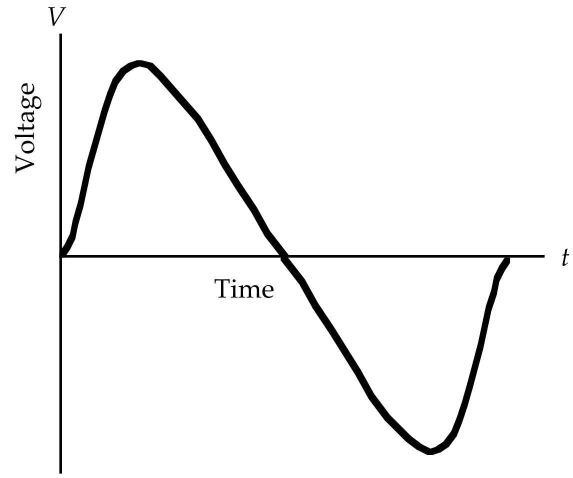
$$-I_1R_1 + I_2R_2 = 0;\quad -I_1(3\text{ k}\Omega) + I_2(6\text{ k}\Omega) = 0.$$

We have three equations for the three unknown currents. When we solve them simultaneously, we get

$$\boxed{I_1 = 0.25\text{ mA},\ I_2 = 0.125\text{ mA},\ I_3 = 0.375\text{ mA}}.$$

79.





While the slider moves from A to B, the current in solenoid S_1 decreases, causing a decrease in the magnetic flux in the core. This decrease in flux generates an induced emf in solenoid S_2, which opposes the decrease and depends on the mutual inductance: $V = -M\,\mathrm{d}I/\mathrm{d}t$.
The time dependence of $\mathrm{d}I/\mathrm{d}t$ is determined by the specific motion of the slider and the self-inductance of solenoid S_1. We assume a smooth motion, with starting and stopping regions, that will give a maximum rate of change of the current a short time after starting. Note that, as the resistance increases, the rate of the fractional change in the resistance will decrease. When the motion of the slider turns around to return to A, the current and the flux will go through a minimum; the induced emf will be zero. Then the flux will start to increase, and the sign of the voltage will change. Assuming the same type of motion, the voltage will be the reverse of the first stage.

80. Before the switch is opened, there is a constant current in the battery loop and no current in the other loop:

$I_1 = \mathcal{E}/R_1 = (6\text{ V})/(6\ \Omega) = \boxed{1.0\text{ A}}$; $I_2 = 0$.

The flux through L_2 is determined by the mutual inductance:

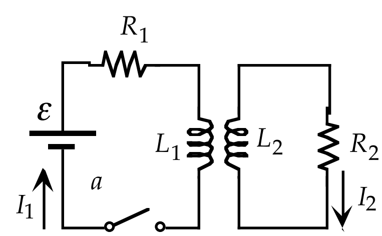
$\Phi_{B21} = MI_1 = (0.7\text{ mH})(1.0\text{ A}) = \boxed{0.7\text{ mWb}}$.

When the switch is opened, the induced emf in L_2 wants to maintain this flux. At $t = 0$, the initial current in L_2 is

$I_{20} = \Phi_{B2}/L_2 = (0.7\text{ mWb})/(30\text{ mH}) = \boxed{0.023\text{ A}}$.

This current will decrease exponentially:

$I_2 = I_{20}\,e^{-R_2 t/L_2} = (0.023\text{ A})\,e^{-(6\ \Omega)(18\text{ ms})/(30\text{ mH})} = \boxed{0.63\text{ mA}}$.



81. We assume that the magnetic field due to the primary winding of the torus is constant:

$B_1 = \mu n_1 I_1$,

so the flux from the primary winding through the N_2 turns of the secondary winding is

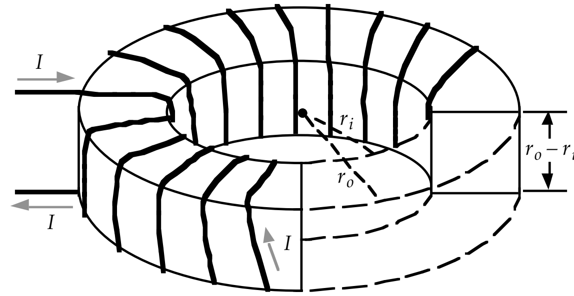
$\Phi_{B21} = N_2 B_1 A = N_2 \mu n_1 I_1 A$.

The mutual inductance is

$$M = \Phi_{B21}/I_1 = N_2 \mu n_1 I_1 A/I_1 = \mu N_2 N_1 A/\ell_1$$
$$= 2500(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(40\text{ turns})(220\text{ turns})(4 \times 10^{-4}\text{m}^2)/(0.35\text{ m})$$
$$= \boxed{3.2 \times 10^{-2}\text{ H}}.$$

The iron core increases B and thus also the linked flux and M, and concentrates flux in the core so that a difference in cross-sectional area is not important.

82. (a) From the symmetry, we know that the magnetic field inside the torus is circular. We apply Ampere's law to a circular path, with $r_i < r < r_o$, to find the magnetic field inside the torus:



$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$;

$B 2\pi r = \mu_0 NI$, which gives $B = \boxed{\mu_0 NI/2\pi r}$.

(b) The energy density of the magnetic field is

$u_B = \frac{1}{2}B^2/\mu_0 = \frac{1}{2}(\mu_0 NI/2\pi r)^2/\mu_0 = \boxed{\mu_0 N^2 I^2/8\pi^2 r^2}$.

(c) Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell, with radius $r_i < r < r_o$, thickness dr, and height $h = r_o - r_i$. The magnetic energy within the torus is

$$U_B = \int u_B \, dV$$

$$= \int_{r_i}^{r_o} \frac{\mu_0 N^2 I^2}{8\pi^2 r^2} h 2\pi r \, dr = \frac{\mu_0 N^2 I^2 h}{4\pi} \int_{r_i}^{r_o} \frac{dr}{r} = \frac{\mu_0 N^2 I^2 (r_o - r_i)}{4\pi} \ln\left(\frac{r_o}{r_i}\right).$$

(d) We find the self-inductance by equating the magnetic field energy to the energy stored in the inductor:

$U_B = U_L$;

$(\mu_0 N^2 I^2/4\pi)(r_o - r_i)\ln(r_o/r_i) = \frac{1}{2}LI^2$, which gives $L = \boxed{(\mu_0/2\pi)N^2(r_o - r_i)\ln(r_o/r_i)}$.

83. Let the capacitance of the original capacitor be C_1 and the inductance of the inductor be L. Then

$1/(LC_1)^{1/2} = \omega_0 = 1.2 \times 10^6$ rad/s.

When the second capacitor of capacitance C_2 is added then equivalent capacitance of the circuit becomes $C = C_1 C_2/(C_1 + C_2)$, and the corresponding new angular frequency is

$1/(LC)^{1/2} = [(C_1 + C_2)/LC_1 C_2]^{1/2} = \omega_1 = 1.6 \times 10^6$ rad/s.

Also, when the capacitor is replace by a resistor of resistance R (= 0.02 Ω) we have an RL circuit, with

$I = I_0 e^{-Rt/L}$, or $e^{-Rt/L} = I/I_0 = 1/2$ at $t = 3.5$ ms.

Solve for L from the last equation above to obtain

$L = Rt/\ln 2 = (0.02\ \Omega)(3.5\text{ ms})/\ln 2 = \boxed{0.1\text{ mH}}$, so

$C_1 = 1/L\omega_0^2 = 1/[(0.1 \times 10^{-3}\text{ H})(1.2 \times 10^6\text{ rad/s})^2] = \boxed{3.9\ \mu\text{F}}$.

Plug these into the second equation above to find $C_2 = \boxed{5.0\ \mu\text{F}}$.

84. The induced emf in the two circuits are given by

$\mathcal{E}_1 = L_1(dI_1/dt) + M(dI_2/dt)$ and

$\mathcal{E}_2 = L_2(dI_2/dt) + M(dI_1/dt)$, so

$dU = I_1\mathcal{E}_1\,dt + I_2\mathcal{E}_2\,dt$

$= L_1 I_1\,dI_1 + M I_1\,dI_2 + L_2 I_2\,dI_2 + M I_2\,dI_1 = d(\frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2)$; and thus

$U = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$.

Suppose $M > (L_1 L_2)^{1/2}$, then we may write

$M = (L_1 L_2)^{1/2} + \Delta M$, where $\Delta M > 0$. We now rewrite U as

$U = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2 = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + [(L_1 L_2)^{1/2} + \Delta M] I_1 I_2 = \frac{1}{2}(\sqrt{L_1}\, I_1 + \sqrt{L_2}\, I_2)^2 + \Delta M\, I_1 I_2$.

We can always adjust the power supplies that drive the currents in the two circuits such that I_1 and I_2 satisfy $\sqrt{L_1}\, I_1 + \sqrt{L_2}\, I_2 = 0$, whereupon $I_1 I_2 < 0$ and

$U = 0 + \Delta M\, I_1 I_2 < 0$.

This is impossible. Regardless of the value of the currents in the two circuits, once a magnetic field is established, the magnetic energy U associated with it must be positive, since the energy density is proportional to B^2 and can never be negative. So we must ensure $U \geq 0$, which requires that $\Delta M \leq 0$ and

$M = (L_1 L_2)^{1/2} + \Delta M \leq (L_1 L_2)^{1/2}$.